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Theoretical uncertainties in sparticle mass predictions from computational tools

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Abstract: We estimate the current theoretical uncertainty in sparticle mass predictions by comparing several state-of-the-art computations within the minimal supersymmetric standard model (MSSM). We find that the theoretical uncertainty is comparable to the expected statistical errors from the Large Hadron Collider (LHC), and significantly larger than those expected from a future e^+e^- Linear Collider (LC). We quantify the theoretical uncertainty on relevant sparticle observables for both LHC and LC, and show that the value of the error is significantly dependent upon the supersymmetry (SUSY) breaking parameters. We also present the theoretical uncertainty induced in fundamental scale SUSY breaking parameters when they are fitted from LHC measurements. Two regions of the SUSY parameter space where accurate predictions are particularly difficult are examined in detail: the large $\tan\beta$ and focus point regimes.

Keywords: Supersymmetry Breaking, Supersymmetry Phenomenology, Beyond Standard Model, Supersymmetric Standard Model.

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1. Introduction

Weak-scale supersymmetry (SUSY) is well motivated [1, 2] because it solves the technical hierarchy problem, removing ultra-violet quadratic divergent corrections to the Higgs mass. Viable weak-scale supersymmetry implies over a hundred "soft breaking" terms in the MSSM lagrangian at the weak scale, making general analysis intractable. By making model-dependent assumptions on the origin of SUSY breaking for the MSSM fields, one often provides relations between the weak-scale SUSY breaking parameters, vastly reducing the available parameter space. Many theoretical schemes of SUSY breaking exist, the most popular of which can be classified by the mechanism that mediates SUSY breaking from a hidden sector to the MSSM fields. In this paper, we will use minimal supergravity (mSUGRA) [3], minimal anomaly mediation (mAMSB) [4] and minimal gauge (mGMSB) [5] mediation.

Supposing SUSY is discovered at a present or future collider, it will be a major challenge to measure the SUSY breaking parameters with good accuracy. In order to determine the free parameters, physical sparticle masses (or kinematical variables related to them) will be measured. These sparticle masses must be turned into SUSY breaking parameters, typically at some high scale. Provided enough information is collected, this will allow tests on the relations between the high-scale parameters. For example, a simple question already addressed [6]–[9] is: do the gaugino masses unify, and if so, do they unify at the same scale as the gauge couplings? A double affirmative would be strong support in favour of SUSY GUT-type schemes. On the other hand, there are many other possibilities: for example, string models can be non-universal [10], or unify at an intermediate scale.

There is a large literature upon expected empirical errors on the observables both a Linear Collider [11]–[15] and the LHC [16]–[19]. It will be essential to know the theoretical uncertainty if we are to discriminate models of SUSY breaking. There are several studies of how empirical errors propagate into uncertainties on fundamental-scale SUSY breaking parameters in the literature [20, 21, 22, 16, 7, 23, 8, 9].

The deduction of high-scale SUSY breaking parameters from observables inevitably involves theoretical errors coming from the level of approximation used (e.g. neglected higher order terms), and it is these uncertainties that we study here. We use four modern codes to calculate MSSM spectra: ISAJET 7.64 [24], SOFTSUSY 1.71 [25], SPHENO 2.0 [26] and SUSPECT 2.101 [27]. We use the differences in results between the codes to define the current theoretical uncertainties. We have done our best to eliminate differences due to bugs by examining the relevant parts of codes in detail if there was an obvious large discrepancy. However, it would be unrealistic to claim that all of the codes are completely bug free. We therefore take a practical interpretation of 'theoretical uncertainty': after all, when fitting experimental data to SUSY breaking models, one must use one of the available computational tools. The precise implementation of the known higher-order corrections differs and has been found to produce significantly different results (for example, using different scales for parameters in the highest-order corrections). Therefore, certainly the differences in results between the codes are due (at least to a large part) to unknown higher-order corrections, matching more traditional notions of 'theoretical uncertainty'.

Previously, there have been some studies of differences between various calculations of the MSSM spectrum. 2-5% differences in sparticle masses were noticed [28] between various particles along the Snowmass (SPS) [29] model lines 1a and 6 between the SOFTSUSY 1.2, ISAJET 7.51 and SUSPECT 2.0 programs. Model line 2 (the so-called focus point line, with very heavy scalar sparticles) was observed to show huge 30% differences in the masses of the weak gauginos. Differences in the spectra and branching ratios were also observed in [30] comparing ISAJET 7.58, SUSYGEN 3.00 and PYTHIA 6.2. Moreover, large 10% level differences between SOFTSUSY 1.3, ISAJET 7.58 and feynSSG were observed [18] in the mSUGRA Post-LEP benchmarks [31]. Some of the benchmarks in some of the codes were found to not be consistent with electroweak symmetry breaking, unless one fiddled with the Standard Model inputs m_t and $\alpha_s(M_Z)$. These were noticeably points E,F (focus-points) and K,M (high $\tan\beta$ and high $m_0, m_{1/2}$ points). Likewise, the input parameters had to be adjusted in [31] to get similar spectra from ISAJET 7.51 and SSARD. Initial results highlighting the differences between the predictions in the focus-point and high $\tan\beta$ regimes have already been presented by the current authors as a conference proceeding [32]. We will present the main results of this last work here for completeness, updated to state-of-the art calculations.

In this paper, we push these initial observations of theoretical uncertainties further by (i) comparing them to the expected experimental accuracies at the LHC and a future LC and (ii) examining their effect upon future empirical fits to fundamental-scale SUSY breaking schemes. We take expected statistical errors upon sparticle observables from previous studies of mSUGRA at the LHC [16] at two of the LHC benchmark points. We then perform a fit to mSUGRA at each point with each of the four codes. The statistical precision of the fitted fundamental mSUGRA parameters may then be compared with the theoretical error by looking at the differences between the four fits. We next quantify and present the theoretical error in the coloured sparticle masses at the SPS points. We also re-examine the SPS points providing theoretical uncertainties on mass predictions for sparticles that may kinematically be accessed at a 500 GeV LC. Quantification of errors at particular points will give us an idea of their magnitude and whether or not they significantly depend upon the SUSY breaking scheme (or point).

In section 2, we introduce the four state-of-the art calculations we will use and their level of approximation: ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101. 'Tricky' regions of the MSSM parameter space, where it is difficult to make accurate predictions, are discussed in section 3: large $\tan\beta$ and the focus-point regime. In section 4.1, we perform the LHC mSUGRA empirical fits and then quantify the theoretical error on squark and gluino masses for the SPS points. In section 4.2, we quantify theoretical errors on masses relevant for a 500 GeV Linear Collider. Finally, there are conclusions and an outlook in section 5.

2. The codes

We compare the latest versions of four public SUSY renormalisation group evolution (RGE) codes which we think constitute a representative sample of such programs: ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101. The basic principle of the SUSY mass spectrum calculation is the same in all programs: Gauge and Yukawa couplings are taken as input parameters at the electroweak scale. However, in the MSSM, in order to define them in the $\overline{\text{DR}}$ scheme, from experimental observables, one must first subtract threshold corrections from sparticles. The sparticle spectrum is unknown at this stage and so to begin the calculation, some guess is made for the soft SUSY breaking parameters and spectrum. The MSSM parameters are then run to the high scale M_X by RGEs. At M_X, boundary conditions are imposed on the SUSY breaking parameters. Couplings and SUSY parameters are then run back down to $M_{\text{SUSY}} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ where the μ and B MSSM Higgs potential parameters are set in order to give correct radiative electroweak symmetry breaking consistent with an input value of $\tan\beta = v_2/v_1$ (v_1, v_2 being the two Higgs fields' vacuum expectation values (VEVs)). The SUSY masses are calculated and radiative corrections are applied, and the parameters are run down to the electroweak scale. Finally, the whole process is iterated in order to obtain a stable solution.

An overview of which corrections are implemented in each of the four programs is given in table 1. From this table we already expect some differences in the results due to the different levels of radiative corrections applied. In particular ISAJET 7.64 has no finite radiative corrections to most of the sparticle masses. However, we note that even if each column of the table were identical, one could expect different numerical results from each of the codes. In practise, if a quantity is calculated at one-loop, one has the freedom (at one-loop accuracy) of using whatever scale one desires for parameters in the one-loop correction itself. The difference between using, for instance, pole or running masses or couplings derived in the $\overline{\text{MS}}$ or $\overline{\text{DR}}$ scheme in one-loop corrections is formally of higher-

	ISAJET 7.64	SUSPECT 2.101	SOFTSUSY 1.71	SPHENO 2.0
RGEs	2–loop	2–loop scalars at 1–loop	2–loop	2–loop
VEVs	not running	running (1–loop)		
Yukawa cpl.				
h_t	full 1–loop	full 1–loop	full 1–loop	full 1–loop
h_b	full 1–loop	$bg + \tilde{b}\tilde{g} + \tilde{t}\tilde{\chi}^{\pm}$ loops		full 1–loop
Higgs sector				
tadpoles	3rd gen. (s)fermions	complete 1–loop [33]		2-loop [33, 34]
h^0, H^0	1–loop [35]	var. ops.	2–loop [36]	2–loop [37, 38]
SUSY masses				
$\tilde{\chi}^{\pm}, \tilde{\chi}^0$	some corr. for $\tilde{\chi}_1^{\pm}$	1–loop aprox. for ΔM_1, ΔM_2, $\Delta\mu$		full 1–loop
$\tilde{t}$	—	1–loop approx.	full 1–loop	full 1–loop
$\tilde{b}$	—	1–loop approx.	full 1–loop	full 1–loop
$\tilde{g}$	all have 1-loop $g\tilde{g} + q\tilde{q}$ re-summed			

Table 1: RGEs and radiative corrections implemented in ISAJET 7.64, SUSPECT 2.101, SOFTSUSY 1.71 and SPHENO 2.0.

loop order, but leads to non-negligibly different results. Indeed, one can roughly estimate the effects of higher-loop terms by the difference in predicted masses by varying the scale of parameters in the highest loop included.

Let us now point out some of the differences in more detail. A subtle but important issue is the treatment of Yukawa couplings. In general, they are derived from the quark masses as

$$h_t = \sqrt{2}\,\frac{m_t}{v_2}\,, \qquad h_b = \sqrt{2}\,\frac{m_b}{v_1}\,, \tag{2.1}$$

where m_t and m_b are the running top and bottom quark masses in the $\overline{\rm DR}$ scheme. Obviously, it makes a difference whether the VEVs $v_{1,2}$ are running (SOFTSUSY 1.71, SPHENO 2.0, SUSPECT 2.101) or not (ISAJET 7.64). It makes an important difference how the $\overline{\rm DR}$ masses are calculated from the pole or $\overline{\rm MS}$ masses. We first discuss Standard Model threshold corrections, and afterwards the sparticle loop corrections.

- ISAJET 7.64 takes 2-loop QCD corrections to m_t into account, including the shift from the $\overline{\rm MS}$ to the $\overline{\rm DR}$ scheme as [39]

  $$m_t(M_t)^{\overline{\rm DR}}_{\rm SM} = M_t \left[1 + \frac{5\alpha_s}{3\pi} + \left(16.11 - 1.04\left(5 - \frac{6.63}{M_t}\right)\right)\left(\frac{\alpha_s}{\pi}\right)^2\right]^{-1}, \tag{2.2}$$

  where M_t is the top pole mass and $\alpha_s = \alpha_s(M_t)$ at 3-loops in the $\overline{\rm MS}$ scheme. For the bottom quark mass, ISAJET 7.64 takes a hard-coded value of $m_b(M_Z)^{\overline{\rm DR}}_{\rm SM} = 2.82$ GeV [40].

- SOFTSUSY 1.71 and SPHENO 2.0 calculate both h_t and h_b at $Q = M_Z$. The $\overline{\rm DR}$ top mass is related to the pole mass by 2-loop QCD [39, 41]:

  $$m_t(Q)^{\overline{\rm DR}}_{\rm SM} = M_t \left[1 - \frac{\alpha_s}{3\pi}(5 - 3L) - \alpha_s^2\left(0.538 - \frac{43}{24\pi^2}\,L + \frac{3}{8\pi^2}\,L^2\right)\right] \tag{2.3}$$

where $L = \ln(M_t^2/Q^2)$. For the b quark, 3-loop relations [42] are used to calculate $m_b(M_b)^{\overline{\rm MS}}_{\rm SM}$ from the b pole mass, which is run to M_Z by 3-loop RGEs [43, 44]. $m_b(M_Z)^{\overline{\rm MS}}_{\rm SM}$ is then related to the $\overline{\rm DR}$ mass by [25]

$$m_b(M_Z)^{\overline{\rm DR}}_{\rm SM} = m_b(M_Z)^{\overline{\rm MS}}_{\rm SM}\left[1 - \frac{\alpha_s}{3\pi} - \frac{35\alpha_s^2}{72\pi^2} + \frac{3g_2^2}{128\pi^2} + \frac{13g_1^2}{1152\pi^2}\right] \qquad (2.4)$$

in SOFTSUSY 1.71. SPHENO 2.0 neglects the last two terms of eq. (2.4).

- SUSPECT 2.101 uses 2-loop relations and 2-loop RGEs to derive $m_b(M_b)^{\overline{\rm MS}}_{\rm SM}$ and $m_t(M_t)^{\overline{\rm MS}}_{\rm SM}$ from the b and t pole masses. For the conversion to the $\overline{\rm DR}$ scheme, eqs. (2.3) and (2.4) are applied (with $L = 0$ in eq. (2.3) since $Q = M_t$).
  Using $Q = M_t$ to define $m_t(Q)$ (ISAJET 7.64, SUSPECT 2.101) is in principle more accurate than using $Q = M_Z$ (SOFTSUSY 1.71, SPHENO 2.0) since then the QCD logs between M_Z and M_t are re-summed. We refer the reader to the respective manuals [24]–[27] for more details.

The next step is to include SUSY loop corrections. For m_t, all four programs apply full 1-loop SUSY corrections according to [33]. For m_b, the full 1-loop (ISAJET 7.64, SPHENO 2.0) or the leading (SOFTSUSY 1.71, SUSPECT 2.101) SUSY corrections are included. The $\tan\beta$ enhanced corrections to m_b are re-summed as given in [45] in all four programs, see [32]. Still, there are some important differences in the implementation of these corrections. For example, the $\tilde{g}\tilde{t}$ correction to m_t is

$$\begin{aligned}\left(\frac{\Delta m_t}{m_t}\right)^{\tilde{g}\tilde{t}} &= -\frac{\alpha_s}{3\pi}\Big\{B_1\left(m_t^2, m_{\tilde{g}}^2, m_{\tilde{t}_1}^2\right) + B_1\left(m_t^2, m_{\tilde{g}}^2, m_{\tilde{t}_2}^2\right) + \\ &\quad + \frac{m_{\tilde{g}}}{m_t}\sin 2\theta_{\tilde{t}}\left[B_0\left(m_t^2, m_{\tilde{g}}^2, m_{\tilde{t}_1}^2\right) - B_0\left(m_t^2, m_{\tilde{g}}^2, m_{\tilde{t}_2}^2\right)\right]\Big\}. \qquad (2.5)\end{aligned}$$

ISAJET 7.64 adds this correction at m_t, but using a scale $Q = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$, (this is done to avoid double counting of logarithmic corrections which are included via step functions in the RGEs) and $\alpha_s = \alpha_s(m_{\tilde{g}})$ in eq. (2.5), while SOFTSUSY 1.71 and SPHENO 2.0 calculate it at $Q = M_Z$ and SUSPECT 2.101 at $Q = M_t$. Accordingly, $m_t = m_t(Q)^{\overline{\rm DR}}_{\rm SM}$ in the term $m_{\tilde{g}}/m_t$ enters with different values in all four programs. Due to differences in the inclusion of finite radiative corrections to sparticle masses, the gluino and stop masses in (2.5) vary from program to program. In particular, ISAJET 7.64 calculates the corrections to $m_{\tilde{g}}$ at $Q = m_{\tilde{g}}$ while SUSPECT 2.101 calculates them at $Q = M_Z$, which leads to quite different gluino masses. Analogous differences exist in the other contributions to $(\Delta m_t)^{SUSY}$ as well as in the calculation of $(\Delta m_b)^{SUSY}$.

Another comment is in order concerning α_s. The value of $\alpha_s(M_Z)$ from experiment is given in the $\overline{\rm MS}$ scheme. SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101 take $\alpha_s(M_Z)^{\overline{\rm MS}}$ as input and convert it to the $\overline{\rm DR}$ scheme [33]

$$\alpha_s^{\overline{\rm DR}}(M_Z) = \frac{\alpha_s(M_Z)^{\overline{\rm MS}}}{1 - \Delta\alpha_s}, \qquad (2.6)$$

where

$$\Delta\alpha_s = \frac{\alpha_s(M_Z)}{2\pi}\left[\frac{1}{2} - \frac{2}{3}\ln\left(\frac{M_t}{M_Z}\right) - 2\ln\left(\frac{m_{\tilde{g}}}{M_Z}\right) - \frac{1}{6}\sum_{\tilde{q}}\sum_{i=1,2}\ln\left(\frac{m_{\tilde{q}_i}}{M_Z}\right)\right]. \qquad (2.7)$$

In SUSPECT 2.101, the log terms are not added explicitly but included via threshold functions in the RGEs. Also ISAJET 7.64 re-sums the logs in (2.7) via step-function decoupling in the RGEs. The finite term, however, is not taken into account. The difference due to the finite term is small but relevant (1%). One could in principle interpret the input $\alpha_s(M_Z)$ in ISAJET 7.64 as already being the effective Standard Model $\overline{\text{DR}}$ value. Since, however, for some corrections ISAJET 7.64 effectively takes hard-wired values of $\alpha_s(M_Z)^{\overline{\text{MS}}} = 0.118$, we take the canonical input value of $\alpha_s(M_Z)^{\overline{\text{MS}}} = 0.118$ for all codes.

3. Tricky corners of SUSY parameter space

As a general point, when quantifying errors on predicted observables, we assume that the results from the codes follow a gaussian probability distribution. This is *a priori* unjustified, but we prefer it to quoting minimum and maximum values because we believe that the additional information included in the variance is desirable. For example, if three codes all provided identical results and one gave a result further away, we think that the true uncertainty ought to be less than the range of minimum-maximum results.

3.1 Large $\tan\beta$

Large $\tan\beta$ has always been recognised as a difficult case since it requires a thorough treatment of the bottom Yukawa coupling. Figure 1 shows h_b of the four different programs as a function of $\tan\beta$ in the mSUGRA model. We see that SOFTSUSY 1.71 and SPHENO 2.0 agree very well on h_b, and there is also good agreement with ISAJET 7.64. Comparing only these programs we would assign a $\lesssim 3\%$ uncertainty on h_b even for very large $\tan\beta$. The agreement with SUSPECT 2.101 is however not so good, and we find $4-8\%$ uncertainty taking all four programs into account. The effect of re-summing the $\tan\beta$ enhanced SUSY loop corrections can be seen when comparing the solid and dotted lines in figure 1, the solid line being the result of ISAJET 7.64, where the $(\Delta m_b)^{SUSY}$ corrections are resummed, and the dotted one being the result of ISASUSY 7.58, where this re-summation is not applied.

The bottom Yukawa coupling has its largest effect in the Higgs sector when it is large (at high $\tan\beta$): the evolution of $m^2_{H_1}$ is driven by h_b,

$$\frac{dm^2_{H_1}}{dt} \sim \frac{3}{8\pi^2}\, h_b\, X_b + \ldots\,, \qquad X_b = \left(m^2_{\tilde{Q}} + m^2_{\tilde{D}} + m^2_{H_1} + A^2_b\right), \qquad (3.1)$$

where $t = \ln Q$, Q being the renormalisation scale. Differences in $m^2_{H_1}$ directly translate into the physical Higgs boson masses since

$$m^2_A = \frac{1}{c_{2\beta}}\left(\overline{m}^2_{H_2} - \overline{m}^2_{H_1}\right) + \frac{s^2_\beta\, t_1}{v_1} + \frac{c^2_\beta\, t_2}{v_2} - M^2_Z\,. \qquad (3.2)$$

Here $\overline{m}^2_{H_i} = m^2_{H_i} - t_i/v_i$, $i = 1, 2$, and $t_{1,2}$ are the tadpole contributions. The self energies of Z and A have been neglected in eq. (3.2).

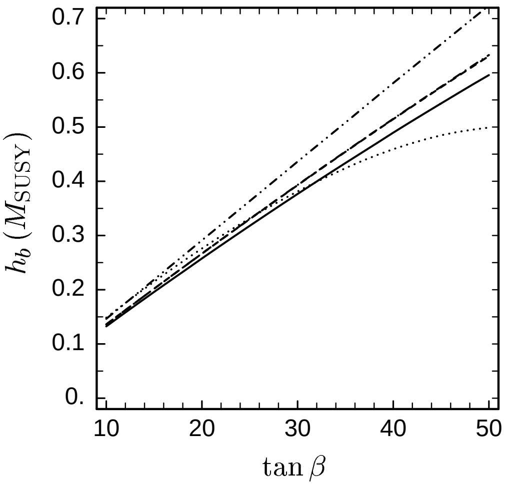

Figure 1: Bottom Yukawa coupling at $M_{\rm SUSY}$ scale as a function of $\tan\beta$, for $m_0 = 400\,$GeV, $m_{1/2} = 300\,$GeV, $A_0 = 0$, $\mu > 0$, $M_t = 175\,$GeV; full (dotted) lines: ISAJET 7.64 (7.58), dashed: SOFTSUSY 1.71, dash-dotted: SPHENO 2.0, dash-dot-dotted: SUSPECT 2.101.

Figure 2 shows the Higgs boson masses obtained by the four programs as a function of $\tan\beta$ (the results obtained by ISAJET 7.58 are again shown as dotted lines in figure 2). Let us first discuss the masses of A^0 and $H^{\pm}$. For $\tan\beta = 10-50$, we find differences in m_A and $m_{H^{\pm}}$ of about $10-50\,$GeV, dominated over most of the $\tan\beta$ range by the difference between SPHENO 2.0 and SUSPECT 2.101. This has to be compared with differences of 100 GeV and more encountered with earlier versions, see for instance the dotted lines representing ISAJET 7.58. Assuming the error to be gaussian, we now have $\Delta m_{A,H^{\pm}} \simeq \pm 10\,$GeV at $\tan\beta = 25$ and $\Delta m_{A,H^{\pm}} \simeq \pm 20\,$GeV at $\tan\beta = 50$. The bottom Yukawa coupling is, however, not the only source of differences in m_A. Another source is, for example, whether one uses running or pole values for masses in the calculation of one-loop tadpoles $t_{1,2}$. Also, the scale and scheme of parameters in the one-loop expressions for the tadpoles all vary. These differences are formally of higher order and indeed each program has a different approach.

The situation is somewhat different for the (h^0, H^0) system because here additional radiative corrections are necessary. It is well known that these involve a theoretical uncertainty on m_{h^0} of about 3 GeV [46], evidence of which can be seen in figure 2. For completeness we note that SUSPECT 2.101 offers various choices of Higgs mass calculations. In figure 2, we have used its default m_{h^0} routine, i.e. `ichoice(10) = 0`, shown as a black dash-dot-dotted line. If we use instead SUSPECT 2.101 with FeynHiggsFast, `ichoice(10) = 3`, we get $m_{h^0} \sim 115\,$GeV, shown as a grey dash-dot-dotted line in figure 2. This will be relevant later in this paper when we discuss mSUGRA fits to LHC data.

3.2 Focus point

For large m_0, the running of $m^2_{H_2}$ becomes very steep and very sensitive to the top Yukawa coupling:

$$\frac{dm^2_{H_2}}{dt} \sim \frac{3}{8\pi^2}\, h_t\, X_t + \dots\,, \qquad X_t = \left(m^2_{\tilde{Q}} + m^2_{\tilde{U}} + m^2_{H_2} + A_t^2\right). \tag{3.3}$$

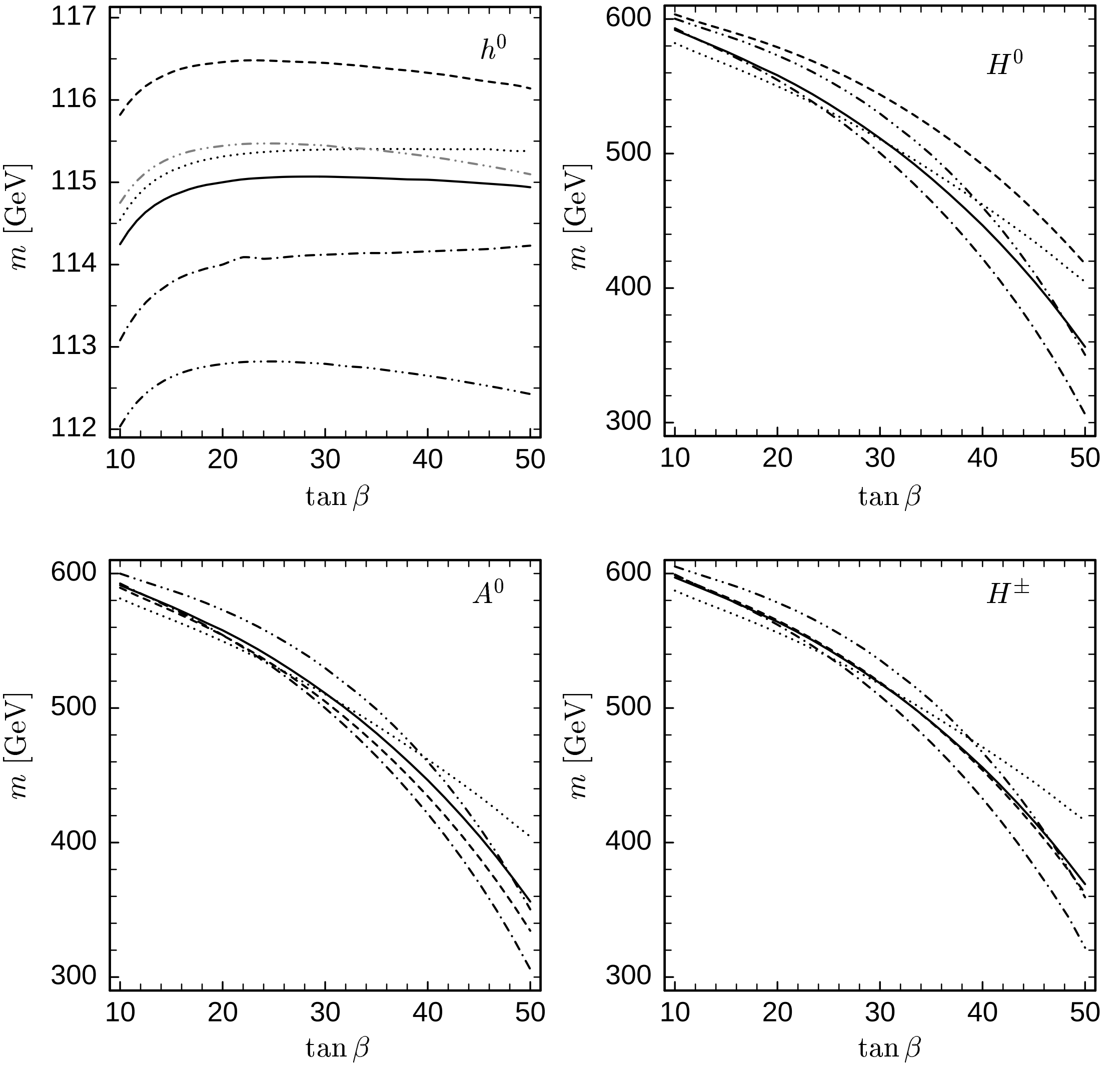

Figure 2: Higgs boson masses as a function of $\tan\beta$, for $m_0 = 400$ Gev, $m_{1/2} = 300\,\mathrm{GeV}$, $A_0 = 0$, $\mu > 0$, $M_t = 175\,\mathrm{GeV}$; full (dotted) lines: ISAJET 7.64 (7.58), dashed: SOFTSUSY 1.71, dash-dotted: SPHENO 2.0, dash-dot-dotted: SUSPECT 2.101 (for h^0, the grey dash-dot-dotted line corresponds to SUSPECT 2.101 + FeynHiggsFast).

As a result, the μ parameter given by

$$\mu^2 = \frac{\overline{m}_{H_1} - \overline{m}^2_{H_2}\tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}\,M_Z^2 \tag{3.4}$$

becomes extremely sensitive to h_t. This is visualised in figure 3 where we show h_t and μ as functions of m_0. The other parameters are $m_{1/2} = 300\,\mathrm{GeV}$, $A_0 = 0$, $\tan\beta = 10$ and $\mu > 0$. There is reasonable agreement on μ up to $m_0 \sim 1\,\mathrm{TeV}$, and the differences observed for $m_0 \lesssim 2\,\mathrm{TeV}$ are phenomenologically not so important. For larger values of m_0 we observe, however, large discrepancies between the four programs. These lead to completely different chargino/neutralino properties for very large m_0, and likewise to

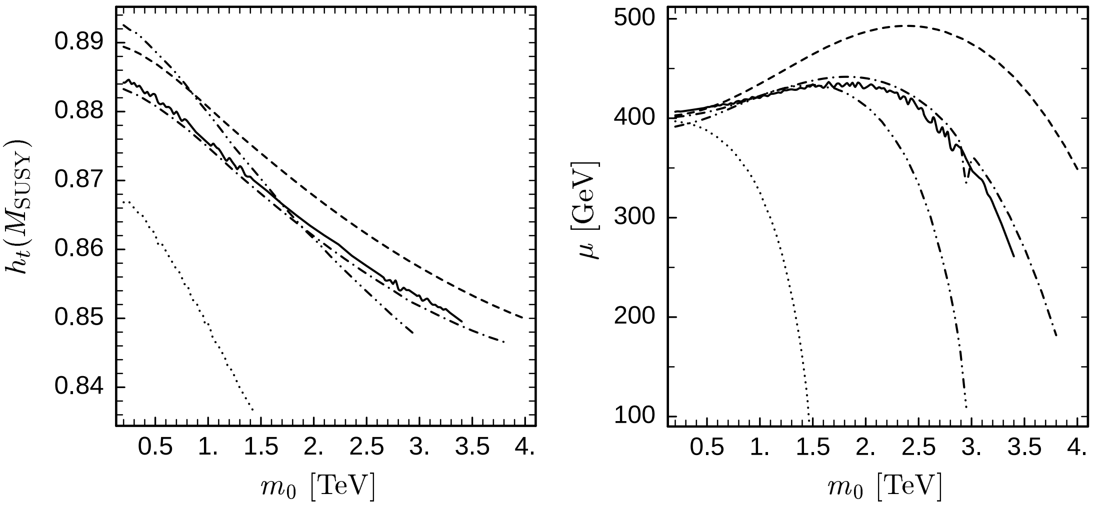

Figure 3: h_t and μ as a function of m_0 for $m_{1/2} = 300\,\mathrm{GeV}$, $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$, $M_t = 175\,\mathrm{GeV}$; full (dotted) lines: ISAJET 7.64 (7.58), dashed: SOFTSUSY 1.71, dash-dotted: SPHENO 2.0, dash-dot-dotted: SUSPECT 2.101.

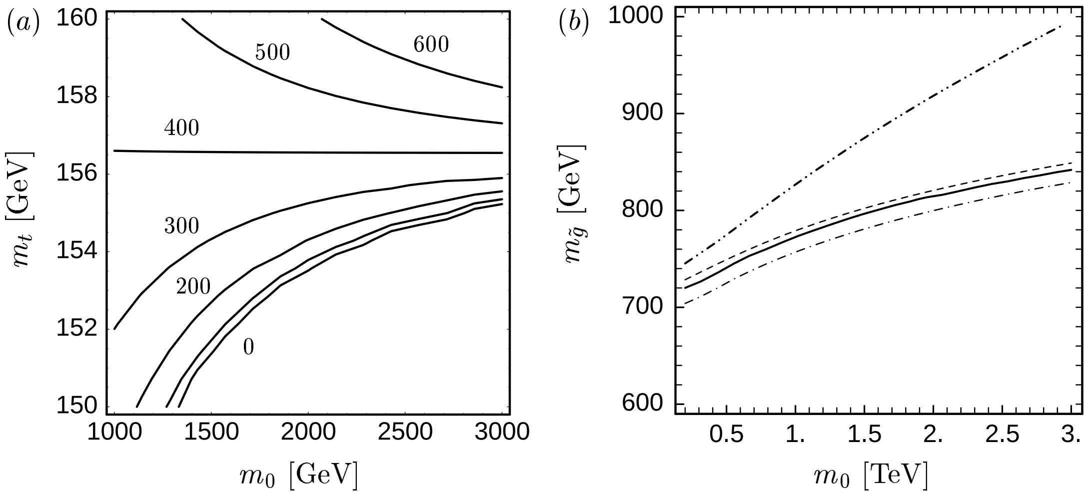

Figure 4: (*a*) $|\mu|$ as given by eq. (3.5) for $m_{1/2} = 300\,\mathrm{GeV}$, $A_0 = 0$, and $\tan\beta = 10$ in the $(m_0, \hat{m}_t)$ plane. (*b*) Gluino mass as a function of m_0 for $m_{1/2} = 300\,\mathrm{GeV}$, $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$, $M_t = 175\,\mathrm{GeV}$; full lines: ISAJET 7.64 dashed: SOFTSUSY 1.71, dash-dotted: SPHENO 2.0, dash-dot-dotted: SUSPECT 2.101.

very different excluded regions, depending on which program is used. The situation is, however, already much better than the one reported in [32], c.f. the dashed lines of ISASUSY 7.58 in figure 3 (earlier versions of the other codes also gave quite different results).

In order to understand the behaviour of μ in figure 3 it is useful to write eq. (3.4) in the form

$$\mu^2 \simeq c_1\, m_0^2 + c_2\, m_{1/2}^2 - 0.5 M_Z^2\,. \tag{3.5}$$

Approximate analytical expressions for c_1 and c_2 can be found e.g., in [47, 48]. For $A_0 = 0$ and $\tan\beta = 10$ we get [48]

$$c_1 \sim \left(\frac{m_t}{156.5\ \mathrm{GeV}}\right)^2 - 1\,, \qquad c_2 \sim \left(\frac{m_t}{102.5\ \mathrm{GeV}}\right)^2 - 0.52\,. \tag{3.6}$$

Figure 4*a* shows contours of constant μ in this approximation in the $(m_0,\, m_t)$ plane. Notice the fast increasing dependence on m_t for increasing m_0. For $m_t \sim 156-157$ GeV, μ becomes almost independent of m_0. This is a signal of the actual focus point behaviour [49], which is defined as the insensitivity of m_{H_2} to its GUT scale value. Eq. 3.4 then implies that μ is insensitive to m_0, since m_{H_1} appears with a suppression factor of $\tan^2\beta - 1$. Interpreting m_t in eq. (3.6) as $m_t(M_{\mathrm{SUSY}}) = h_t(M_{\mathrm{SUSY}})\, v_2(M_{\mathrm{SUSY}})/\sqrt{2}$, we can directly relate the m_0 dependence of μ in figure 3 to that of h_t.

Some more comments are in order. Firstly, SUSPECT 2.101 has only 1–loop RGEs for scalar SUSY parameters. For large m_0, the 2–loop terms lead to $\mathcal{O}(10\%)$ correction and should thus be taken into account. Secondly, as already mentioned in section 2, the sparticle masses that enter the radiative corrections have different values in different codes. In particular, there are large differences in the gluino masses between SUSPECT 2.101 and the other codes, as illustrated in figure 4*b*. Since the gluino mass enters h_t via the $\tilde{g}\tilde{t}$ correction eq. (2.5), this may account for the different slope of h_t (and consequently of μ) in SUSPECT 2.101 compared to the other programs, as evident in figure 3.

It thus seems that for reliable results in the large m_0 region, a more complete calculation of the top Yukawa coupling at the 2–loop level is necessary. This should include finite radiative corrections to all sparticle masses at the full 1–loop level.

4. Comparison of theoretical and experimental uncertainties

4.1 Fits of mSUGRA parameters to LHC data

In the ATLAS TDR [16], a case study was made of fitting the mSUGRA model to possible measurements of six reference scenarios. We have re-analysed these fits for two of these scenarios, LHC Point 1 and Point 2. Here squarks and gluinos are produced with the dominant decays $\tilde{g} \to q\tilde{q}_{L,R}$, $\tilde{q}_L \to \tilde{\chi}^0_2 q \to \tilde{\chi}^0_1 hq$, $\tilde{q}_R \to \tilde{\chi}^0_1 q$. The assumed measurements for the two points for low ($\mathcal{L} = 30\,\mathrm{fb}^{-1}$) and high ($\mathcal{L} = 300\,\mathrm{fb}^{-1}$) luminosity are given in table 2. They were estimated in ref. [16] by using ISAJET 7.34 and simulating the ATLAS experiment to determine expected empirical errors.[1]

With each of the programs under discussion we have performed a χ^2 fit of the mSUGRA parameters m_0, $m_{1/2}$ and $\tan\beta$ to the data of table 2, taking $A_0 = 0$ and $\mu > 0$. The results are listed in tables 3 and 4. The quoted errors are at 1σ (68.3% C.L.) from a simultaneous fit of all three parameters, i.e. $\Delta\chi^2 = 3.53$. In case of SUSPECT 2.101, we have used its default option, `ichoice(10)=0`, for the calculation of the h^0 mass. When linking SUSPECT 2.101 with FeynHiggsFast, `ichoice(10)=3`, the results for m_0 and $m_{1/2}$ practically do not change. However, we get much lower values for $\tan\beta$: $\tan\beta = 1.7 \pm 0.1$ for Point 1, and $\tan\beta = 6.06 \pm 2.06$ for Point 2 at high luminosity.

Quantity	**Low-L**	**High-L**
m_h (Point 1)	$95.4 \pm 1.0\,\mathrm{GeV}$	$95.4 \pm 1.0\,\mathrm{GeV}$
m_h (Point 2)	$115.3 \pm 1.0\,\mathrm{GeV}$	$115.3 \pm 1.0\,\mathrm{GeV}$
$m_{hq}^{\max}$	$758.3 \pm 25\,\mathrm{GeV}$	$758.3 \pm 25\,\mathrm{GeV}$
$m_{\tilde{q}_R}$	$959 \pm 40\,\mathrm{GeV}$	$959 \pm 15\,\mathrm{GeV}$
$m_{\tilde{g}}$	$1004 \pm 25\,\mathrm{GeV}$	$1004 \pm 12\,\mathrm{GeV}$
$m_{\tilde{t}_1}$ (Point 1)	none	$647 \pm 100\,\mathrm{GeV}$
$m_{\tilde{t}_1}$ (Point 2)	none	$713 \pm 100\,\mathrm{GeV}$

Table 2: Possible LHC measurements for Point 1 and Point 2, from [16].

Point 1, Low-L

	$\chi^2_{\min}$	m_0 [GeV]	$m_{1/2}$ [GeV]	$\tan\beta$
ISAJET	0.10	490 ± 135	424 ± 25	1.77 ± 0.21
SOFTSUSY	9.30	280 ± 246	425 ± 32	1.60 ± 0.03
SPHENO	0.02	373 ± 175	436 ± 26	2.10 ± 0.15
SUSPECT	0.32	411 ± 116	410 ± 20	2.08 ± 0.16

Point 1, High-L

	$\chi^2_{\min}$	m_0 [GeV]	$m_{1/2}$ [GeV]	$\tan\beta$
ISAJET	0.57	496 ± 61	424 ± 12	1.77 ± 0.20
SOFTSUSY	11.66	356 ± 78	422 ± 12	1.60 ± 0.03
SPHENO	0.27	370 ± 82	436 ± 12	2.10 ± 0.15
SUSPECT	1.79	422 ± 67	409 ± 13	2.08 ± 0.15

Table 3: Fit to LHC measurements for Point 1.

It is interesting to note that not only the central values but also the size of the errors can be quite different. Similarly, the minimum χ^2, which is a measure of the quality of the fit, can show large variations. To make the comparison easier and to visualise correlations and non-gaussian effects, we show in figures 5 and 6 contours of 68% and 95% C.L. for the fits of Points 1 and 2 in the (m_0, $m_{1/2}$) and (m_0, $\tan\beta$) planes. The values of $\Delta\chi^2$ used for these confidence levels are based upon a simultaneous two-parameter fit, i.e. $\Delta\chi^2 = 2.3$ and 5.99. The third parameter, $\tan\beta$ or $m_{1/2}$, is always fixed to its best-fit value. As one can see, the error ellipses have only little or even no overlap. We therefore conclude that the theoretical uncertainty is about the same size as the statistical one in the fitted quantities.

One might expect that the main source of these differences is the theoretical uncertainty on m_h. This is indeed the case for the determination of $\tan\beta$. However, m_h has only little influence on the fit of m_0 and $m_{1/2}$. This becomes clear when using e.g., SUSPECT 2.101 with different routines for the Higgs mass calculation. In figure 6*a*, we show the results of SUSPECT 2.101 + FeynHiggsFast, `ichoice(10) = 3`, as dashed contours in addition to those obtained with its default Higgs mass routine (`ichoice(10) = 0`, solid contours). In

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[1] It is beyond the scope of the present paper to re-perform the experimental analysis in order to have the numbers in table 2 more up-to-date.

Point 2, Low-L

	$\chi^2_{\rm min}$	m_0 [GeV]	$m_{1/2}$ [GeV]	$\tan\beta$
ISAJET	0.03	523 ± 129	424 ± 24	6.55 ± 2.37
SOFTSUSY	0.08	414 ± 140	419 ± 23	4.65 ± 0.76
SPHENO	0.19	405 ± 167	437 ± 26	$\gtrsim 7$
SUSPECT	0.06	444 ± 114	409 ± 18	$\gtrsim 7$

Point 2, High-L

	$\chi^2_{\rm min}$	m_0 [GeV]	$m_{1/2}$ [GeV]	$\tan\beta$
ISAJET	0.14	521 ± 58	424 ± 11	6.52 ± 2.30
SOFTSUSY	0.33	411 ± 68	419 ± 11	4.63 ± 0.88
SPHENO	1.08	394 ± 80	438 ± 13	$\gtrsim 7$
SUSPECT	0.20	450 ± 64	408 ± 11	$\gtrsim 7$

Table 4: Fit to LHC measurements for Point 2.

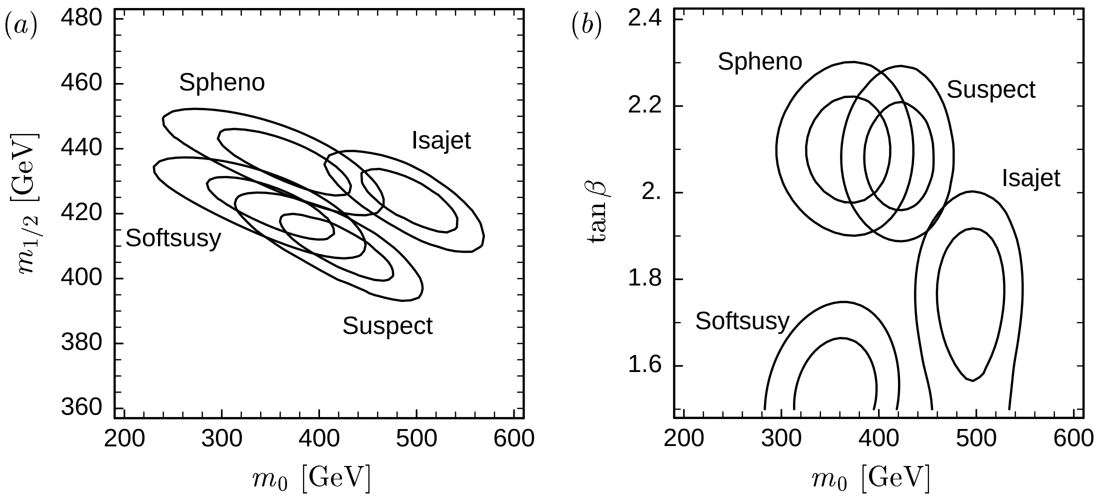

Figure 5: Fit to LHC measurements for Point 1 with ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101 for $\mathcal{L} = 300$ fb^{-1}. Shown are contours of 68% and 95% C.L. for each program in the $(m_0, m_{1/2})$ and $(m_0, \tan\beta)$ planes, with the third parameter fixed to its best fit value, c.f. table 3.

figure 6*b*, we have omitted the default SUSPECT 2.101 results to avoid confusion of the many lines. They would look similar to the SPHENO 2.0 contours but centred at $m_0 = 450$ GeV and without an upper limit on $\tan\beta$. We thus conclude that the uncertainties in m_0 and $m_{1/2}$ mainly come from the differences in the programs pointed out in sections 2–3, and not from different Higgs mass calculations.

We next address the question of how the theoretical uncertainty depends on the SUSY parameter point. Copious quantities of squarks and gluinos are expected to be produced at the LHC, leading to a fairly precise measurement of their masses, particularly if $m_{\chi^0_1}$ is determined accurately by a LC. In table 5 we compare the $\tilde{g}$, $\tilde{u}_L$, $\tilde{u}_R$ and $\tilde{t}_1$ masses

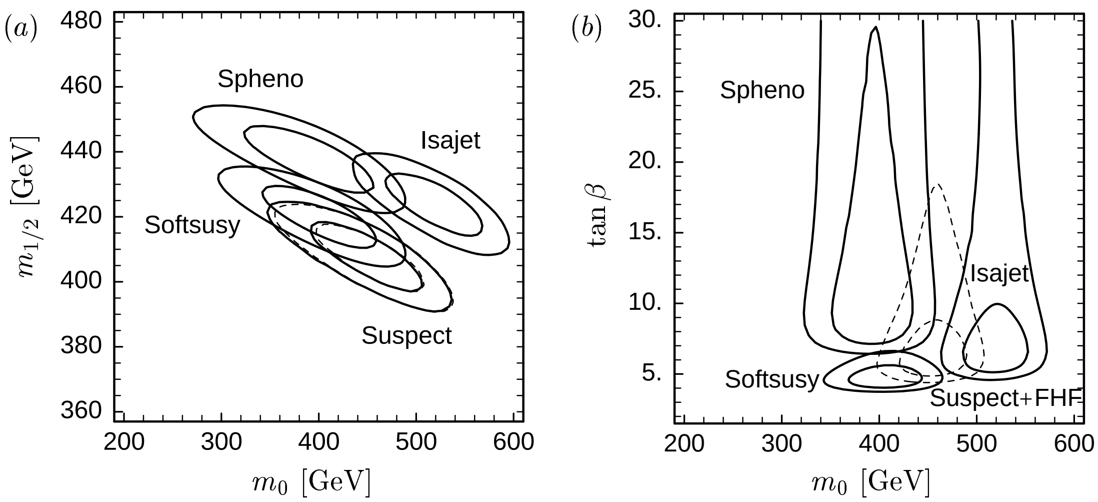

Figure 6: Fit to LHC measurements for Point 2 with ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101 for $\mathcal{L} = 300$ fb^{-1}. Shown are contours of 68% and 95% C.L. for each program in the $(m_0, m_{1/2})$ and $(m_0, \tan\beta)$ planes, with the third parameter fixed to its best fit value, c.f. table 4. In case of SUSPECT 2.101, the solid lines are for its default m_h routine and the dashed lines for m_h calculated with FeynHiggsFast.

obtained by the four programs for the Snowmass (SPS) points [29]. Assuming a gaussian distribution, we quote the variance of these masses as the theoretical error, i.e. $\delta m_X = \sqrt{\frac{1}{N-1}\sum_i [(m_X)_i - \overline{m}_X]^2}$ where $\overline{m}_X$ is the mean of $(m_X)_i$ and $N = 4$ in our case. We make the following observations: (i) the absolute theoretical uncertainty (in GeV) varies from point to point; (ii) the typical relative uncertainty in mSUGRA and mGMSB scenarios in generic (i.e. not tricky) regions of parameter space is about 2 – 5%; (iii) in some cases, in particular in focus point and mAMSB scenarios, the relative uncertainty is larger, about 5 – 10%; (iv) in any case, the theoretical error is of the same order of magnitude as the experimental one.

4.2 Linear Collider measurements

At a high-luminosity e^+e^- Linear Collider, one expects to measure chargino, neutralino and slepton masses with accuracies at the per-cent or even per-mill level [11]–[15]. We thus take the differences in these masses accessible at $\sqrt{s} = 500$ GeV as a measure of the theoretical uncertainty. Tables 6 and 7 compare masses obtained by the four programs for the various SPS points. Only those masses kinematically accessible at a 500 GeV LC are shown. The error is again defined as the variance of the masses as in the previous section.

It turns out that the uncertainty in the LSP mass in mSUGRA and mGMSB scenarios is typically a few hundred MeV, depending on the parameter point. An exception is the focus point scenario (SPS2) where $\delta m_{\tilde{\chi}^0_1} = 1.4$ GeV. For $\tilde{\chi}^0_2$ and $\tilde{\chi}^\pm_1$, we find uncertainties of about 1 – 3 GeV in mSUGRA and mGMSB scenarios. (Here note that with earlier program versions, especially with ISASUSY 7.51 – 7.63, we had discrepancies of 50% and more for focus point scenarios.) For the sleptons we find typical uncertainties of 1 – 2 GeV. We note

mass	code	1a	1b	2	3	4	5	6	7	8	9
$\tilde{g}$	ISAJET	607	936	794	932	732	719	718	944	835	1296
	SOFTSUSY	614	949	802	946	743	730	729	964	852	1306
	SPHENO	594	917	782	914	719	705	704	940	836	1232
	SUSPECT	626	964	870	959	761	730	742	986	902	1395
	error	**13**	**20**	**40**	**19**	**18**	**12**	**16**	**22**	**31**	**67**
$\tilde{u}_L$	ISAJET	536	835	1532	817	730	642	640	858	1079	1233
	SOFTSUSY	549	851	1582	831	753	657	662	876	1083	1291
	SPHENO	565	876	1563	859	764	676	674	910	1127	1314
	SUSPECT	570	886	1595	867	775	681	680	910	1138	1502
	error	**15**	**23**	**27**	**23**	**19**	**18**	**18**	**26**	**30**	**116**
$\tilde{u}_R$	ISAJET	520	807	1529	788	714	622	626	830	1033	1242
	SOFTSUSY	569	884	1592	866	774	681	679	914	1142	1297
	SPHENO	548	847	1552	828	746	655	659	880	1080	1266
	SUSPECT	550	852	1585	832	754	656	662	880	1092	1492
	error	**20**	**32**	**29**	**32**	**25**	**24**	**22**	**35**	**45**	**114**
$\tilde{t}_1$	ISAJET	379	633	947	621	523	236	476	774	951	998
	SOFTSUSY	398	658	974	645	544	232	497	813	987	951
	SPHENO	398	658	964	646	545	248	497	813	982	986
	SUSPECT	410	676	1004	663	560	243	513	831	1015	1140
	error	**13**	**18**	**24**	**17**	**16**	**7**	**15**	**24**	**26**	**83**

Table 5: Gluino and squark masses in GeV for the SPS benchmark points, and their theoretical uncertainties. The theoretical uncertainty is displayed in bold type face and is calculated as described in the text.

that for SPS5, the lighter stop would be accessible, with $m_{\tilde{t}_1} = 235.6$, 232.3, 248.4 and 242.6 GeV for ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101, respectively, corresponding to an error of 7 GeV, c.f. table 5. In the mAMSB scenario, SPS 9, we have much larger uncertainties of ~ 8 GeV for $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_1^+}$. This is due to the fact that SOFTSUSY 1.71 has 2-loop GUT-scale boundary conditions for mAMSB, while the other programs have only 1-loop boundary conditions. If we enforce 1-loop boundary conditions for all four programs the error decreases to 3 GeV. We therefore expect that if all programs were to use 2-loop boundary conditions the error would be better than 3 GeV.

By comparing table 5 with tables 6, 7 we see that the present theoretical uncertainty in mass predictions is significantly smaller for weakly interacting sparticles than for strongly interacting ones. This is expected, since the former have smaller threshold corrections. However, the errors in tables 6, 7 are still larger by up to an order of magnitude than the expected experimental accuracies at an e^+e^- Linear Collider. Moreover, the differences in the masses produce cross sections and branching ratios that differ by a few per-cent. While this is not a problem for determining SUSY-breaking parameters at the low scale, it will be relevant when relating them to GUT-scale parameters in order to test their unification and to determine the sources of SUSY breaking.

5. Conclusions

If sparticles are detected at future colliders, measurements of their properties (and confirmation that they are in fact sparticles) will take place. The question following from this

point	code	$M_{\chi_1^0}$	$M_{\chi_2^0}$	$M_{\chi_1^\pm}$	$M_{\tilde{\nu}_e}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{e}_R}$	$M_{\tilde{e}_L}$	$M_{\tilde{\tau}_1}$	$M_{\tilde{\tau}_2}$
1a	ISAJET	96.6	176.8	176.4	185.8	184.9	142.9	201.9	133.3	206.0
	SOFTSUSY	96.4	178.2	177.6	188.7	187.9	144.9	204.3	136.3	207.8
	SPHENO	97.6	182.9	181.3	190.5	189.6	143.9	206.6	134.6	210.3
	SUSPECT	97.4	179.8	179.1	188.5	187.5	144.9	204.4	135.7	208.1
	error	**0.3**	**1.5**	**1.2**	**1.1**	**1.1**	**0.6**	**1.1**	**0.8**	**1.0**
1b	ISAJET	161.2	299.2	–	–	–	–	–	195.2	–
	SOFTSUSY	161.3	303.5	–	–	–	–	–	204.6	–
	SPHENO	163.7	310.2	–	–	–	–	–	195.1	–
	SUSPECT	162.6	305.5	–	–	–	–	–	199.7	–
	error	**0.7**	**2.6**	**–**	**–**	**–**	**–**	**–**	**2.6**	**–**
2	ISAJET	120.2	221.7	221.5	–	–	–	–	–	–
	SOFTSUSY	118.6	232.0	231.8	–	–	–	–	–	–
	SPHENO	123.8	233.6	232.1	–	–	–	–	–	–
	SUSPECT	123.1	233.0	232.6	–	–	–	–	–	–
	error	**1.4**	**3.2**	**3.1**	**–**	**–**	**–**	**–**	**–**	**–**
3	ISAJET	160.5	297.0	–	–	–	178.3	287.0	170.6	289.2
	SOFTSUSY	160.8	300.4	–	–	–	182.0	290.4	175.3	292.3
	SPHENO	162.7	307.2	–	–	–	180.2	290.3	172.4	292.4
	SUSPECT	161.7	302.4	–	–	–	182.1	290.7	174.8	292.6
	error	**0.6**	**2.5**	**–**	**–**	**–**	**1.0**	**1.7**	**1.3**	**1.6**
4	ISAJET	119.3	219.6	219.6	–	–	–	–	–	–
	SOFTSUSY	118.6	223.6	223.4	–	–	–	–	–	–
	SPHENO	121.4	228.9	227.4	–	–	–	–	–	–
	SUSPECT	121.1	225.9	225.7	–	–	–	–	–	–
	error	**0.8**	**2.3**	**1.9**	**–**	**–**	**–**	**–**	**–**	**–**
5	ISAJET	119.8	225.9	225.9	244.3	242.3	191.5	256.3	180.9	257.7
	SOFTSUSY	118.7	229.4	229.2	248.2	246.1	193.5	259.4	183.7	260.7
	SPHENO	121.1	230.0	229.8	248.0	245.9	192.0	259.4	181.7	260.9
	SUSPECT	120.7	230.6	230.4	247.8	245.5	193.5	259.5	182.8	260.6
	error	**0.6**	**1.2**	**1.2**	**1.1**	**1.0**	**0.6**	**0.9**	**0.7**	**0.9**

Table 6: Differences in predicted masses in GeV for the SPS points 1a to 5 (mSUGRA points). Only sparticles with masses that kinematically can be produced at a 500 GeV Linear Collider are displayed. The theoretical uncertainty is displayed in bold type face and is calculated as described in the text.

line of investigation will be: what can we learn about SUSY breaking from these measurements? Is there a unification of certain SUSY breaking parameters, and is it possible to distinguish various SUSY breaking models?

Vital ingredients for answers to these questions will be the amount and precision of empirical measurements made at the particular colliders [11, 13, 14, 15, 16, 18]. Another vital ingredient will be the precision with which we can relate the experimentally observed quantities at the TeV scale with the fundamental physics at the high-energy scale. Accurate extrapolations require multi-loop results for the RGEs and the related threshold corrections. In this paper, we have addressed the question: what is the current theoretical uncertainty associated with determining fundamental-scale SUSY parameters? To this end, we compared four public state-of-the-art MSSM spectrum calculations: ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101, taking the spread of their results as a

point	code	$M_{\chi_1^0}$	$M_{\chi_2^0}$	$M_{\chi_1^\pm}$	$M_{\tilde\nu_e}$	$M_{\tilde\nu_\tau}$	$M_{\tilde e_R}$	$M_{\tilde e_L}$	$M_{\tilde\tau_1}$	$M_{\tilde\tau_2}$
6	ISAJET	190.0	218.1	215.7	–	–	236.8	–	227.8	–
	SOFTSUSY	190.2	220.6	217.9	–	–	240.9	–	232.8	–
	SPHENO	191.2	225.3	222.3	–	–	237.6	–	229.1	–
	SUSPECT	190.2	222.2	219.6	–	–	240.9	–	232.2	–
	error	**0.3**	**1.7**	**1.6**	**–**	**–**	**1.3**	**–**	**1.4**	**–**
7	ISAJET	162.4	268.0	–	248.7	248.3	127.3	261.1	119.9	263.3
	SOFTSUSY	163.6	263.5	–	247.2	246.9	126.4	259.3	120.5	261.2
	SPHENO	163.4	271.1	–	251.6	251.3	131.0	265.3	123.8	267.3
	SUSPECT	163.6	262.2	–	246.9	246.6	127.8	259.4	121.6	261.4
	error	**0.3**	**2.4**	**–**	**1.2**	**1.2**	**1.2**	**1.6**	**1.0**	**1.6**
8	ISAJET	137.4	254.6	–	–	–	175.7	–	168.9	–
	SOFTSUSY	138.4	261.2	–	–	–	175.4	–	169.9	–
	SPHENO	139.2	266.1	–	–	–	180.3	–	173.5	–
	SUSPECT	140.0	263.5	–	–	–	177.6	–	171.8	–
	error	**0.6**	**2.8**	**–**	**–**	**–**	**1.3**	**–**	**1.2**	**–**
9	ISAJET	174.8	–	175.0	–	–	–	–	–	–
	SOFTSUSY	196.7	–	196.7	–	–	–	–	–	–
	SPHENO	168.0	–	168.4	–	–	–	–	–	–
	SUSPECT	167.3	–	167.3	–	–	–	–	–	–
	error	**7.9**	**–**	**7.9**	**–**	**–**	**–**	**–**	**–**	**–**

Table 7: Differences in predicted masses in GeV for the SPS points 6 to 9 (6: non-minimal SUGRA, 7+8: mGMSB, 9: mAMSB). Only sparticles with masses that kinematically can be produced at a 500 GeV Linear Collider are displayed. The theoretical uncertainty is displayed in bold type face and is calculated as described in the text.

measure of the to-date uncertainty. Although this does not correspond to the usual notion of 'theoretical uncertainty', it is pragmatic in the sense that (at least) one of the available calculational tools will be used to perform fits if and when the relevant data arrives. The uncertainty was shown to be largest in certain tricky corners of parameter space: the focus point region and high $\tan\beta$. However, even in these regions, comparison with previous versions of the codes [32] shows that the theoretical uncertainty has significantly improved. Sparticle masses in these regions are particularly sensitive to the values of the Yukawa couplings (especially the top Yukawa for the focus point, and the bottom Yukawa for the high $\tan\beta$ regime). Slightly different treatments of top and bottom masses can lead to large differences in mass predictions. It is therefore critical that the accuracy in deriving the running top and bottom $\overline{\mathrm{DR}}$ Yukawa couplings is at a maximum in any calculation.

We used previous LHC estimates of expected empirical errors at two benchmark points to perform separate fits to mSUGRA with the four different codes. The parameters resulting from the four calculations show a difference comparable to the statistical error upon them, showing that theoretical uncertainties must be taken into account. We then went on to quantify the theory uncertainty associated with squark and gluino masses for the SPS benchmark points. The theory uncertainty was also quantified for parts of the MSSM spectrum that would be kinematically accessible to a 500 GeV Linear Collider for the SPS points. The tables of spectra listed in the present article may be used for further comparisons with other (possibly new or private) computations. The theoretical uncertainties for

LHC and Linear Collider observables were shown to significantly depend upon the point in parameter space being considered. It will not be very precise, therefore, to use the theory uncertainties calculated here when analysing actual data, since the SUSY breaking parameter point is *a priori* unknown.

If sparticles *are* measured, the correct modus operandi is clear when analysing data: any fits to a fundamental SUSY breaking model should be performed with several state-of-the-art spectrum calculations in order to deduce the level of theoretical uncertainty, which must then be included in final quoted errors. The theory errors must clearly be included when discussing the accuracy required to distinguish between SUSY models on sparticle observables, such as in ref. [50].

We note that each of the codes used assumed a pure MSSM desert up until the GUT scale $\sim 10^{16}$ GeV. We know already [51, 52] that this assumption is called into question because the gauge couplings themselves require $\sim \mathcal{O}(1\%)$ corrections at the high scale in order to make them unify properly. It is thought that this level of correction is perfectly reasonable in, for example, SUSY GUTs since model-dependent GUT-scale threshold corrections [53, 54] are expected from (for example) heavy coloured triplets. If the combined theoretical and empirical accuracy is significantly better than 1%, then observables at colliders could be used to *measure* these threshold corrections.

Fortunately, theoretical errors in sparticle mass predictions are certainly not static. There has been much progress reducing them recently (especially in the Higgs and electroweak symmetry breaking sectors, see for example [38, 46, 55, 56, 34]). We expect this trend to continue, which, as our present results indicate, is desirable if we are to disentangle SUSY breaking from experimental observables.

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